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Qian Li[a]; Jiancai Zhao[b]; Bo Zhao[a]; Xunsheng Zhu[a]
[a] School of Mechanical Engineering, Shanghai Jiao tong University, Shanghai, P. R. China [b] State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai, P. R. China

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Parameter Optimization of Rubber Mounts Based on Finite Element Analysis and Genetic Neural Network

QIAN LI[1], JIANCAI ZHAO[2,*], BO ZHAO[1] and XUNSHENG ZHU[1]

[1]*School of Mechanical Engineering, Shanghai Jiao tong University, Shanghai, P. R. China*
[2]*State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai, P. R. China*

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The structures of vehicle rubber mounts cannot be optimized with conventional optimization methods due to their complex structures and irregular sections. A parameter optimization methodology for a rubber mount based on Finite Element Analysis (FEA) and Genetic Neural Network models is proposed in this study. A FEA model of the rubber mount was developed and analyzed using the software MSC.MARC, and the primary stiffness of rubber mounts with different geometric parameters in three principle directions were obtained by this FEA method. Then the FEA results were used as samples to train the neural network (NN) model which defines the non-linear global mapping relationship between the rubber mount's geometric parameters and its primary stiffness in three principle directions. The fitness values of the population in the genetic algorithm (GA) were calculated by the trained NN model and the optimal solution was acquired with the mutation of population. Finally, experiments were made to validate the reliability of the optimal solution. The proposed optimization method can shorten the product design cycle and decrease the design and trial-product cost considerably.

Keywords: Rubber mount, parameter optimization, finite element analysis, genetic algorithm, neural network

1 Introduction

Mounts are important vibration isolation parts widely used in vehicles. The vibration isolation characters of mounts can greatly affect the vibration's transmission to compartments and affect vehicles' noise, vibration and harshness (NVH) characters. There are mainly two kinds of mounts, rubber and hydraulic damped engine mounts, used in vehicles today. The shapes of rubber components are flexible and the stiffness in different directions can be selected randomly. So rubber mounts have the characteristics of spatial springs which can bear loads in different directions. Rubber mounts can also utilize the damping produced by their internal friction to absorb the vibration and impact energy. Rubbers bond well and easily with metals, and this can simplify the supporting structures and decrease the masses of rubber mounts. So rubber mounts have the advantage of simple processing, low cost and convenient for usage and maintenance; thus rubber mounts are still the most widely used vibration isolation parts in vehicles (1). Nevertheless, rubber components have been designed by experience or experiment, in most instances, because of their complex structures and irregular sections. Considering the waste of design time and trial-product cost resulting from these approaches, more systematic and analytical approaches are desirable.

Many scholars have carried out related research on the parameter optimization of rubber mounts. Jenkins (2) investigated the application of a genetic algorithm in the optimization of structural design. An initial population of designs was generated by stochastic processes and then principles of natural selection and survival of the fittest were applied to improve the designs. Kim and Kim (3) introduced an optimum shape design process of rubber engine mounts using a parametric approach. An optimization code was developed to determine the shape to meet the stiffness requirements of engine mounts, coupled with a commercial non-linear finite element program. Zhao et al. (4) investigated the fatigue crack problem of a rubber mount by theoretical calculation and experimental analysis. Modifications were made to the structure parameters and rubber material of the rubber mount based on the analysis of the FEA results. The stress concentration of the rubber mount at the rubber and metal interfaces was improved and the fatigue life of the improved rubber mount was increased. Beijers et al. (5) established a numerical cylindrical vibration isolator model with the finite element package ABAQUS. Chen et al. (6) studied the structure of a rubber

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[*] Address correspondence to: Jiancai Zhao, State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao tong University, Shanghai 200240, P. R. China. Tel: +86-21-34206316; Fax: +86-21-34206008; E-mail: liq@sjtu.edu.cn

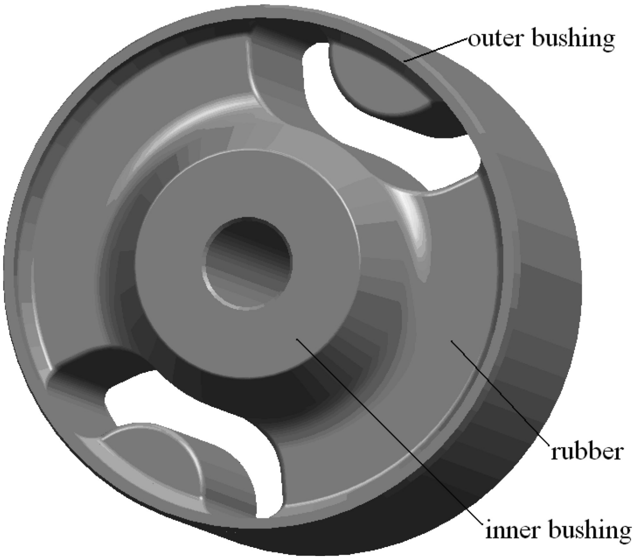


Fig. 1. The three-dimensional diagram of a rubber mount.

mount, factors influencing the stiffness of the assembled rubber mounts were found and improvement solutions were presented. Nevertheless, how to prepare the samples conveniently and specify the structures and dimensions of rubber mounts to meet the design requirements automatically, has not been explicitly discussed in the research.

2 Experimental

A bush type rubber mount used in a passenger car is considered in this study. Figure 1 shows a three-dimensional diagram of this rubber mount. To specify the structure and dimension of the rubber mount to meet the design requirement automatically, a parameter optimization methodology for the rubber mount based on FEA and a genetic neural algorithm was proposed in this study. The FEA method was applied to prepare the samples to train the NN model and the trained NN model was utilized to calculate the fitness values of the population in the GA. Then the optimal solution was obtained by the mutation of population in the GA.

2.1 The Parameter Optimization Methodology for the Rubber Mount Based on FEA and a Genetic Neural Network

The parameter optimization methodology for the rubber mount based on FEA and genetic neural network can be described as follows:

1. Specify design parameters, objective function and the feasible ranges of design parameters and develop the optimization model for the rubber mount.
2. Develop the FEA model of the rubber mount and analyze the model, obtain the primary stiffness of rubber mounts with different structures in three principle directions.
3. Use the FEA results as the samples to train the error back propagation (BP) NN model between the rubber mount's geometric parameters and its primary stiffness in the three principle directions
4. Utilize the trained NN model to calculate the fitness values of the population; principles of natural selection and survival of the fittest in GA were applied to search for the optimal solution.
5. Sample rubber mounts of the optimal solution were produced and experiments to measure the stiffness of these sample mounts were made to validate the reliability of the optimization methodology.

Figure 2 shows the basic idea of the optimization methodology.

2.2 Optimization Model and Analysis

In most applications, it is required that the primary stiffness of rubber mounts in three principle directions meet specific values (3). Figure 3 shows the real shape of the rubber

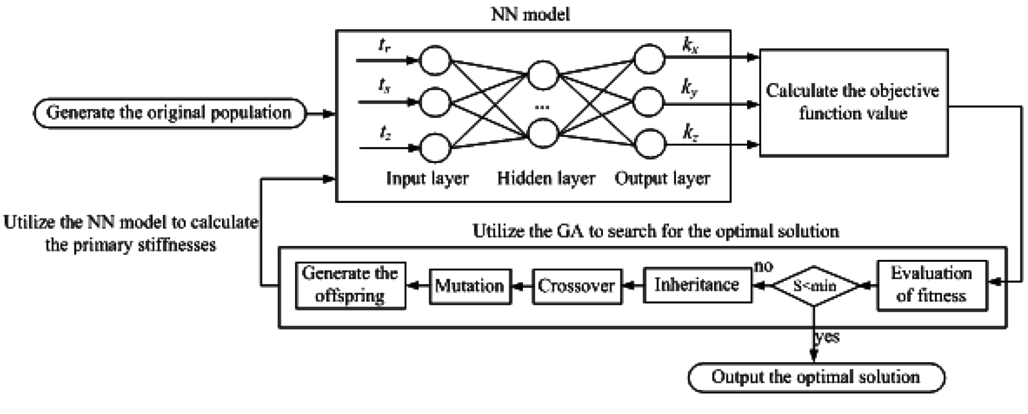
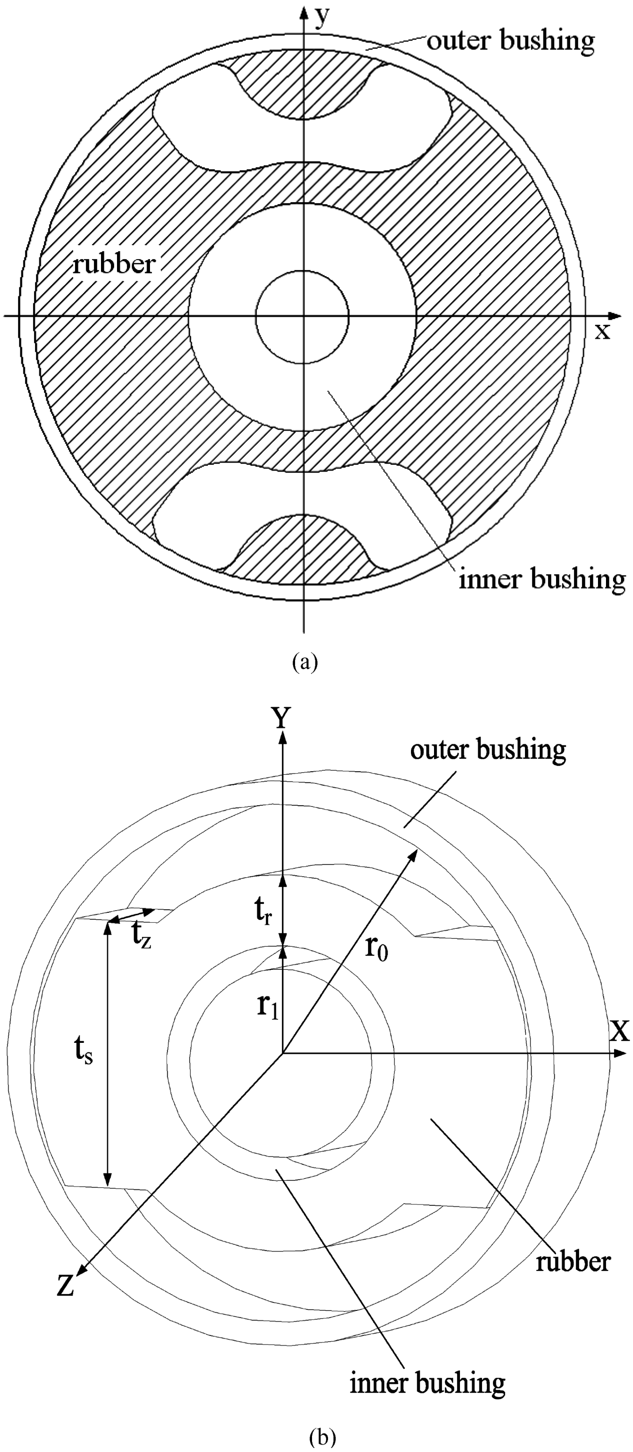


Fig. 2. The procedure of the parameter optimization method.



(a)

(b)

Fig. 3. Schematic diagram of the rubber mount and characterized geometry for optimization. (a) General shape; (b) Characterized shape for optimization.

mount and the characterized geometry of it. For convenience, the axes of local coordinates shown in Figure 3(a) will be used throughout this study. There are five geometric parameters to define the shape of the bush type rubber mount, as shown in Figure 3(b). Among them, r_1 and r_0 are given by the layout design. Therefore, three parameters t_r, t_s and t_z are used as design parameters in this study. Once the design parameters are chosen, the object of parameter optimization is to minimize the following objective function F.

$$F = \lambda_1\left(k_x - k_x^{des}\right)^2 + \lambda_2\left(k_y - k_y^{des}\right)^2 + \lambda_3\left(k_z - k_z^{des}\right)^2 \quad (1)$$

Where k_x^{des}, k_y^{des} and k_z^{des} are the primary stiffness in x, y and z directions, respectively, and λ_1, λ_2 and λ_3 are weighing factors of these three stiffnesses. The superscript *des* indicates the desired primary stiffness values determined in the system vibration analysis. The weighing factors can be adjusted based on the significance of the stiffness values. In view of dynamic response in the vibration system, the stiffness in the x direction is the most important and the stiffness in the y and z directions are of less importance (3). Hence, a high weight factor corresponding to the stiffness in the x direction and low factor in the y and z directions were chosen. λ_1, λ_2 and λ_3 were determined as 1, 0.5 and 0.5, respectively. Based on the system vibration analysis, the desired dynamic stiffness k_x^{des}, k_y^{des} and k_z^{des} are 521N/mm, 175N/m and 137N/mm, respectively. So the optimization expression is rewritten as Equation 2.

$$F = (k_x - 521)^2 + 0.5(k_y - 175)^2 + 0.5(k_z - 137)^2 \quad (2)$$

If the design is feasible, the values of the design variables must be in certain ranges. Based upon design specification, the ranges of design variables are given as follows:

$$\begin{aligned} 2\,\text{mm} &\le t_r \le 10\,\text{mm} \\ 10\,\text{mm} &\le t_s \le 40\,\text{mm} \\ 30\,\text{mm} &\le t_z \le 45\,\text{mm} \end{aligned} \quad (3)$$

2.3 Preparation of the Samples for Training the NN Model

Samples are needed to train the NN model which defines the non-linear global mapping relationship between the rubber mount's geometric parameters and its primary stiffness in three principle directions. To acquire the stiffness of rubber mounts with different structures, the most customary way is to produce rubber mounts with different structures and then measure the stiffness; this is a waste of

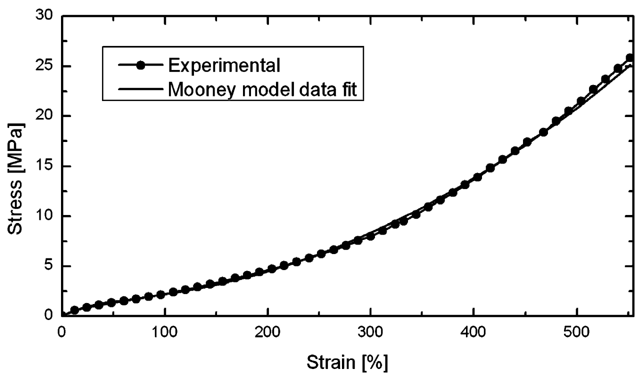


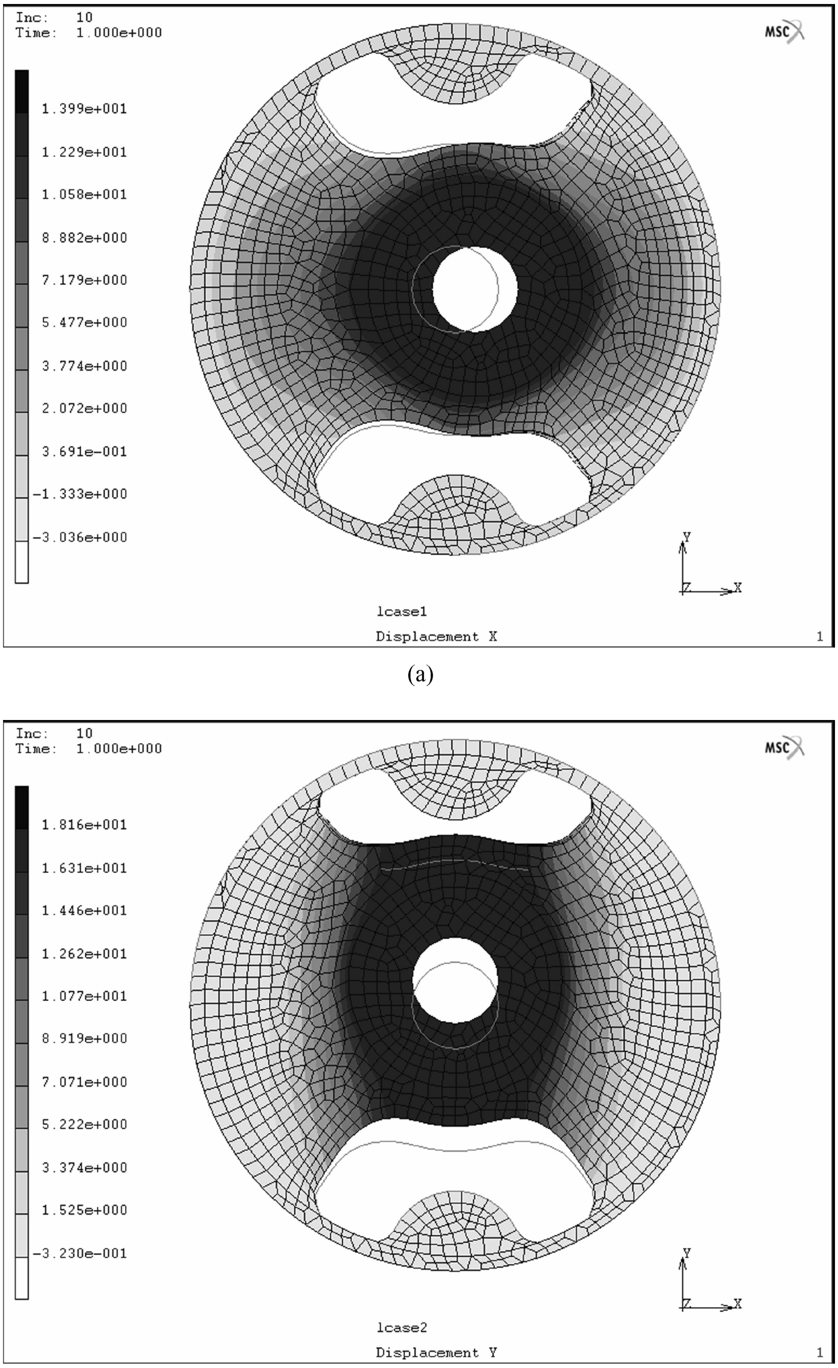
Fig. 4. The stress vs. strain curves of the rubber material.

time and cost. Hence, a FEA method to predict the stiffness of the rubber mount is proposed in this study.

2.4 The Hyperelastic Constitutive Model of the Natural Rubber Material

The natural rubber can be considered as a hyperelastic material, showing highly non-linear elastic isotropic behavior with incompressibility (7). A relationship between stress and strain in the hyperelastic material, generally characterized by strain energy potentials, is essential for the FEA of rubber components.

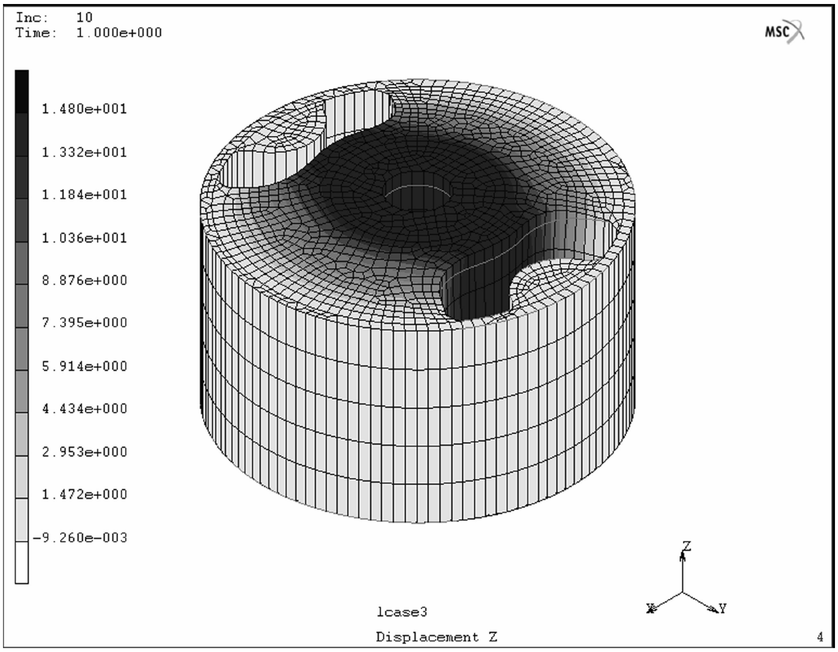
The three parameter Mooney-Rivlin function was selected to specify the constitutive model of the natural rubber material in this study. The three parameter Mooney-Rivlin model can be expressed as follows



(a)

(b)

Fig. 5. (a) The deformed shape and displacement contour at load in x direction; (b) The deformed shape and displacement contour at load in y direction; (c) The deformed shape and displacement contour at load in z direction. (*Continued*)



(c)

Fig. 5. (Continued)

(8, 9).

$$W = C_{10}(I_1 - 3) + C_{01}(I_1 - 3) + C_{11}(I_1 - 3)(I_2 - 3) \quad (4)$$

Where W is the strain energy potential of the rubber material. I_1, I_2 and I_3 are the first order, second order and third order invariable strain values, respectively. The moduli C_{10}, C_{01} and C_{11} can be determined from uniaxial tensile tests of the natural rubber material. By data fit using the data acquired in uniaxial tensile tests we determined that C_{10}, C_{01} and C_{11} are -0.438, 1.537 and 0.132, respectively. Figure 4 shows the stress vs. strain curves of the experimental data and the Mooney model data fit.

2.5 FEA of the Rubber Mount

The model of the rubber mount was analyzed by the finite element analysis software MSC.MARC. The model consists of three distinct material domains, the first domain is the outer bushing with material Al, Si, Mn, Zn, second is the middle layer with rubber material, and third is the inner bushing with material AlMg3. These different material domains were considered perfectly bonded to one another. The four node quad element was selected for the model and the Advanced Front Quad mesh tool was selected to generate elements automatically (10, 11). Figures 5 (a), (b) and (c) illustrate the deformed shapes and displacement contours of the rubber mount in the x, y and z directions, respectively. The primary stiffness can be calculated as the ratios of loads to maximum displacements.

2.6 Preparation of the Samples

The orthogonal test method was adopted to design the geometric parameters of the samples. The level numbers of parameters t_s, t_r and t_z were 5, 4 and 4, respectively, so the orthogonal experiment table $L_{20}(5 \times 4^2)$ was selected (12). The primary stiffness of the rubber mounts with different parameters wascalculated by the above FEA method. The geometric parameters of the samples and their stiffness, k_x, k_y and k_z, are listed in Table 1.

2.7 Training of the NN Model

A three layer BP NN ($3 \times 7 \times 3$) was adopted to train the model; the neurons number of the input layer was 3, which corresponds to the three design parameters t_r, t_s and t_z, the neurons number of the hidden layer was set as 7, the neurons number of the output layer was 3, which corresponds to the primary stiffness of the rubber mount in the three principle directions, k_x, k_y and k_z. Figure 6 shows the sum squared errors of the different epochs. It can be seen from Figure 6 that the errors decrease with the process of the training. The training was set to end when the error was smaller than 0.02 and the weight factors between the neurons were saved to calculate the fitness the values of population in the GA.

2.8 The Optimization Using GA

Principles of natural selection and survival of fittest in GA were used to search for the optimal solution. The population size, crossover rate, mutational rate, and fitness

Table 1. The samples prepared and their stiffness

Sample No.	t_s *(mm)*	t_r*(mm)*	t_z*(mm)*	k_x *(N/mm)*	k_y *(N/mm)*	k_z *(N/mm)*
1	10	2	45	220.93	67.3	57.5
2	10	5	30	120.67	40.34	38.15
3	10	8	35	161.28	53.61	48.86
4	10	10	40	250.03	70.7	62.67
5	17.5	2	40	435.23	94.43	88.21
6	17.5	5	45	465.18	112.17	105.12
7	17.5	8	30	293.65	80.37	72.82
8	17.5	10	35	293.65	105.27	97.26
9	25	2	35	363.3	137.92	111.1
10	25	5	40	472.18	153.07	130.8
11	25	8	45	512.21	163.85	136.12
12	25	10	30	427.48	102.45	95.19
13	32.5	2	30	454.62	142.72	119.72
14	32.5	5	35	507.75	163.15	123.43
15	32.5	8	40	562.78	189.52	135.33
16	32.5	10	45	586.85	198.25	143.48
17	40	2	45	602.61	209.87	148.92
18	40	5	30	521.71	165.11	124.28
19	40	8	35	600.77	201.47	134.38
20	40	10	40	648.97	212.27	146.84

function of the GA were selected as 100, 0.6, 0.09 and $1/F$, respectively. When the variation of the fitness values was smaller than 0.1, the mutation finished and the optimal solution was output and saved.

3 Results and Discussion

3.1 Results

The obtained optimal parameters, t_r, t_s and t_z, were 6.4, 37.8, and 32.6 mm, respectively using the above optimization method combining FEA and Genetic Neural Network.

3.2 Experiment Validation

The stiffness of the optimal rubber mounts was measured on a MTS 810 elastomer test system to determine whether it was consistent with the design requirement. Six sample rubber mounts of the optimal solution were produced and fixed on the upper and bottom pole of the test system by a custom clamp. The bottom pole was fixed on the base of the test system and a load cell was installed on the upper pole. Recording the displacements of the rubber mount at specific loads in the x, y and z directions, the ratios of loads to displacements were regarded as stiffness. The average measured stiffness in the x, y and z directions were 534 N/mm, 168 N/mm and 145 N/mm; the relative errors between the measured stiffness and the desired stiffness were 2.5%, 4.0%, and 5.8%, respectively.

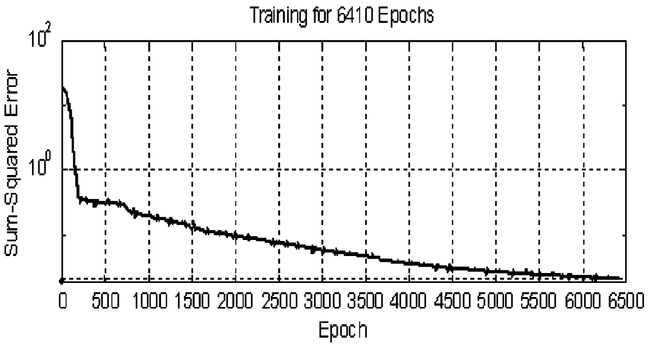


Fig. 6. The variation of sum square errors.

4 Conclusions

Through a combination of FEA and Genetic Neural Network methods, the parameters of the rubber mount were optimized to meet the design requirements conveniently and automatically. The samples to train the NN were prepared by the FEA method without first producing the practical rubber mounts with different structures. The mutation of population in the genetic neural network was used to search for the optimal solution automatically. The stiffness of the optimal rubber mount met well with the design requirement. Relative errors between the measured stiffness of the optimal solution and the desired stiffness in three principle directions were 2.5%, 4.0%, and 5.8%, respectively. The proposed optimization method can shorten the product design cycle, decrease the design and trial-product cost remarkably and this method can be used to optimize parameters of any bushing type rubber mount.

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